Stochastic Modeling and Validation of Three-Tier Supply Chains Using Multiple Tools

Alok K. Verma
Old Dominion University
averma@odu.edu

Abstract

Efficient and effective supply chain management assists an organization in getting the right goods and services to the right place at the right time, in the proper quantity, and at acceptable cost. Managing this process involves developing and overseeing relationships with suppliers and customers, controlling inventory, and forecasting demand, all requiring constant feedback from every link in the chain. The base stock model is applied to a three-tier, single-product supply chains to calculate order quantities and reorder point at various locations within the supply chain. A computer-based discrete event simulation model using ProModel software is used to study the three-tier supply chain and to validate the results from stochastic models. Results indicate that agility of supply chains can be enhanced by using the stochastic models to calculate order quantities and reorder points. In addition to reducing the total cost of inventory, probability of backorder and customer dissatisfaction is minimized. Results are further validated with physical simulations. Both computer-based simulation and physical simulation demonstrate the improvement in the agility of the supply chain with reduced cost for inventory.

Introduction

Inventory management throughout the supply chain is critical when the demand is not deterministic. Demand variability increases as one moves up the supply chain, away from the customer and any small changes in customer demand can result in large variation in orders upstream. This phenomenon is known as Bullwhip effect. Thus, it is necessary to study inventory models for uncertain demand. Wallace and Spearman [5] and Zheng [7] have contributed major work on statistical modeling of production and inventory control. Wilson breaks the inventory control problem into two distinct parts: determining the order quantity, which is the amount of inventory that will be produced with each replenishment, and determining the reorder point or the inventory level at which replenishment will be triggered. Cachon and Zipkin [8] emphasized backorder policies in a multistage supply chain where the base stock inventory model is used.

A survey was conducted to identify key issues related to supply chain facing the ship-building industries under a project of NSRP. The key issues are long lead-time, inventory cost, scheduling problems, irregular performance, challenges in synchronizing flow with suppliers, and vendors furnishing information late. Wincel [6] introduces lean methodology as the key factor in its supply chain strategies. Issues related to streamlining supply chain are discussed by Copacino and Slagmulder [3] and Cooper [2]. Inventory issues in supply chain are explored further by Handfield and Nichols [4] and Ayers [1].

We have considered the virtual company with a three-tier supply chain. We applied the base stock inventory model at the primary supplier, secondary supplier, and the warehouse. We calculated the fill rate (the probability that the order has arrived before demand for each case) and calculated reorder points at the primary supplier, secondary supplier, and the warehouse for five replenishment lead-times (12, 8, 6, 4, and 2 months) using a mathematical model. Physical and discrete event simulations were run to validate the optimum inventory levels and reorder point at warehouse, primary supplier, and secondary supplier. Positive validation was obtained by both methods.

The Base Stock Model

The base stock model uses a continuous time frame and makes the following assumptions:

1. Demands occur one at a time.
2. Any demand not filled from stock is backordered.
3. Replenishment lead-times are fixed and known.
4. Replenishments are ordered 1 at a time.
5. Products can be analyzed individually.

We used the following notations:

l	Replenishment lead-time (in years)
x	Demand during replenishment lead-time (in units), a random variable
G (x)	P (X<=x), cumulative distribution function of demand during replenishment lead-time; we will allow G to be continuous or discrete.
Θ	E [X] = mean demand (in units) during lead-time l
r	reorder point which represents the inventory level that triggers a replenishment order
R	r + 1 base stock level
S	r - Θ, safety stock level

The base stock model is equivalent to the Japanese *kanban* system (with *kanba*n size of 1), since the order quantity is 1.

Application Runs of Base Stock Model to Three-Tier Supply Chain

Replenishment lead-time = 12 months

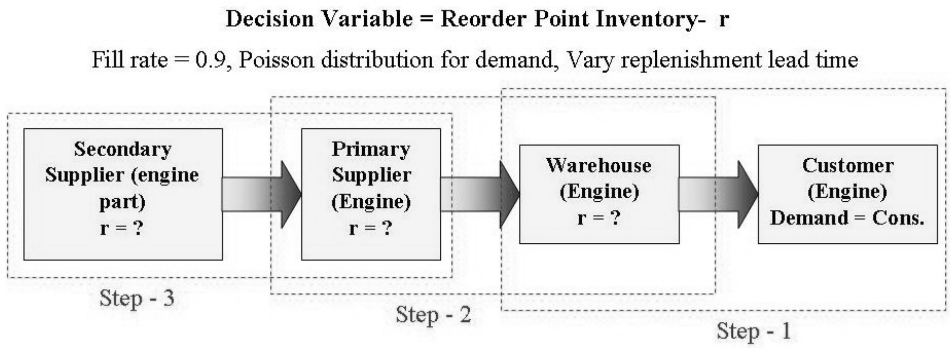

Figure 1. Supply chain considered for base stock model

At the warehouse, demand during 12 months is 10 units /year, and average demand is 10 units per year.

Results from Base Stock Model

Table 1 summarizes all the results for the base stock model and frequency of order. Order cost is assumed to be $25 per order. The total cost is calculated by using

$$TC = c\,(Q/2 + r - \Theta) + \text{Order cost} \quad (1)$$

Total Cost vs. Replenishment Lead-Time

The total inventory cost is plotted against replenishment lead-time in Figure 2.

Table 1. Summary of results of costs (base stock model)

Replenishment Lead-time*	Warehouse ($)	Primary Supplier ($)	Secondary Supplier ($)
12	925	1175	1450
8	741.25	925	1175
6	775	925	1225
4	725.5	975	1350
2	316.25	450	650

* Months

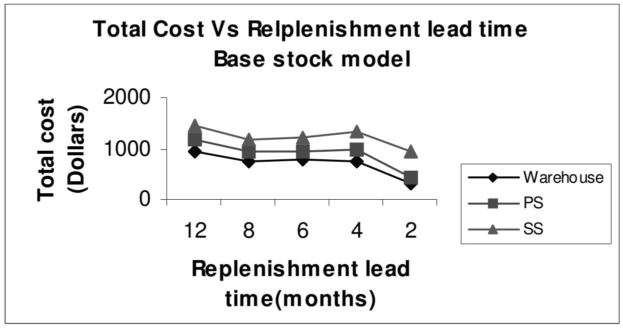

Figure 2. Total cost vs. replenishment lead-time (base stock model)

Reorder Point vs. Replenishment Lead -Time

The reorder point decreases with replenishment lead- time. Reorder point is plotted against replenishment lead-time in Figure 3.

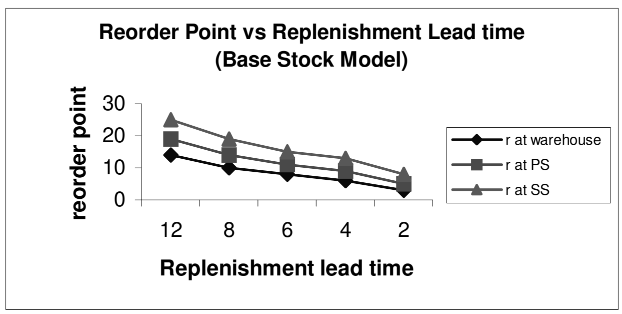

Figure 3. Reorder point vs. replenishment lead-time (base stock model)

Summary

The graph in Figure 3 shows the decreasing trend in the reorder point from warehouse to secondary supplier for the same lead-time. The total inventory cost decreases with replenishment lead-time for the base stock model. We can conclude from Figure 2 that there is decreasing trend in costs of the warehouse, primary supplier, and secondary supplier for the same replenishment lead-time. The base stock model emphasizes an order quantity of 1 and can be used where demand is stochastic. The base stock model proves to be better for small lead-time.

Physical Simulation of the Base Stock Model

Primary goal of conducting the physical simulation is to validate the results obtained from the mathematical models. Physical simulations are being used very effectively as a teaching tool for lean training. This physical simulation models a three-tier, single-product supply chain. ABC Company uses a certain type of engine for their product. The company's Final

Assembly Department withdraws these engines from the warehouse as needed. The warehouse receives engines from the primary supplier, which receives the engine parts, like cylinders, from the secondary supplier. We will make the assumption that only 1 cylinder is needed per engine. We are interested in inventory levels at the warehouse, primary supplier, and secondary supplier.

The movement of parts is shown in Figure 4. Engines are pulled from warehouse based upon a demand that follows Poisson distribution.

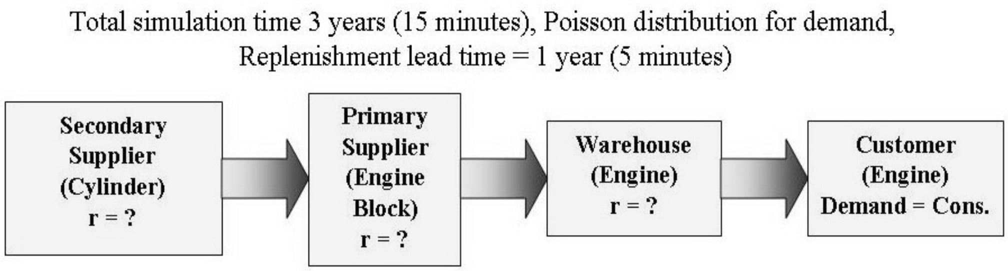

Figure 4. Layout of supply chain for physical simulation

Simulation Activity Time Frame

The total duration of simulation for each phase is 15 minutes (3 years). The customer sends the order requirement form to the warehouse at the start of simulation. Inventory at the warehouse goes below reorder point when the customer demands parts from the warehouse (at first minute). The warehouse then sends the order requirement form to the primary supplier. This triggers production activity at the primary supplier, which has a replenishment lead-time of one year. Replenishment lead-time at the secondary supplier is also one year. The warehouse has initial inventory (equal to reorder point). Customer demand is satisfied with this initial inventory.

In second year, the primary supplier sends the parts to the warehouse as per the schedule provided by the warehouse. Demand at the warehouse also follows a Poisson distribution. When the inventory level at the primary supplier goes below the reorder point (at 6th minute), it sends the order requirement form to the secondary supplier. This initiates production at the secondary supplier.

In third year, the secondary supplier starts sending parts to the primary supplier (at 11th minute). The primary supplier sends the engine to the warehouse as per the schedule received in the second year. The warehouse fulfills the customer demand as per the order requirement form provided by the customer in the third year.

Simulation Phases

During Phase 1, the amount of initial inventory is the same as the reorder point calculated but lower than the quantities predicted by the mathematical model. The level of inventory is 10 at the warehouse, 14 at the primary supplier, and 19 at the secondary supplier. Customer demand is 10 units per year. These values are intentionally kept lower than the ideal values of inventory predicted by the mathematical model.

Any demand not filled from stock is backordered. The number of backorders during this phase is noted in the form provided at each department. Simulation activity takes place and data are collected. The base stock model assumes a replenishment quantity of 1 unit. Hence, there is a single piece flow in the supply chain.

Inventory at the end of the simulation at the warehouse, primary supplier, and secondary supplier is documented. The ideal values calculated by the mathematical model are warehouse=14, primary supplier=19, secondary supplier=25. The total number of backorders is documented and results are shown in a spreadsheet.

During Phase 2, inventory levels are kept at the optimum values predicted by the mathematical model. The inventory levels are the same as the reorder points in this phase, too. With optimum levels of inventory, no backorders were documented in this phase, confirming the results predicted by mathematical models.

During Phase 3, the inventory levels are kept intentionally higher than the optimum levels, and the reorder points are as shown in the figure below. No backorders were observed in this phase due to high inventory levels, but inventory costs were high due to large inventory level.

Distribution of Demand

We ensure that the demand at the warehouse, primary supplier, and secondary supplier follows Poisson distribution as in the case of the mathematical models. This is done by using Stat-Fit software to calculate demand quantities for the customer, primary supplier, and secondary supplier. The values obtained are shown in Table 2.

Table 2. Order quantity vs. replenishment lead-time

Demand at Customer	**Demand at Primary Supplier**	**Demand at Secondary Supplier**
2	3	4
3	4	5
2	3	4
2	2	3
1	2	3
10	**14**	**19**

Performance Metrics

The assumptions about backorder cost and inventory holding costs match with the mathematical models. It is assumed that each backorder costs $100, and unit inventory holding cost is $20. The order cost is assumed to be $25 per order. In base stock model, the order quantity is 1; therefore, the total numbers of orders are same as the order quantity. Table 3 shows the data, which is collected in a spreadsheet:

Table 3. Performance metrics

Performance Criteria	Phase I	Phase II	Phase III
Total number of orders	24	33	44
Order cost	$600.00	$825.00	$1,100.00
Excess Inventory	6	24	41
Total number of backorders	10	0	0
Cost of each backorder ($)	$100.00	$100.00	$100.00
Total cost of backorder	$1,000.00	$0.00	$0.00
Cost of inventory cost	$10.00	$10.00	$10.00
Excess Inventory cost	$60.00	$240.00	$410.00
TOTAL COST	$1,660.00	$1,065.00	$1,510.00

Summary

Excess inventory and the number of backorders is documented at the end of each phase. The inventory holding cost and the backorder cost are calculated in each phase. Ten backorders were observed during Phase 1 because of inadequate inventory at the warehouse. Therefore, the total backorder cost is $1,000 in Phase 1. During Phase 3, excess inventory exists, and the cost associated with this inventory is $410.

Phase 2 includes the optimum level of inventory as predicted by the mathematical models. In this case, backorder cost is 0, and excess inventory cost is higher than Phase 1 but lower compared with Phase 3. The total cost of inventory is the lowest in Phase 2, as predicted by the mathematical models. The physical simulation used Lego blocks for engine blocks, cylinders, and assembled engines.

Discrete Event Simulation

The primary goal of computer-based simulation is to demonstrate that the base stock model can effectively predict the level of inventory at the reorder point. Another goal is to compare the results obtained here with those of mathematical model and the physical simulation model. Discrete event simulation is a pedagogical tool that uses computer models to study a production system with the goal of optimizing its performance. ProModel simulation software is used for analyzing and assessing the flow of parts through a two-tier supply chain system. The model uses four locations to indicate the key players in the supply chain, namely, the customer, warehouse, primary supplier, and secondary supplier. The layout of the model is shown in Figure 5.

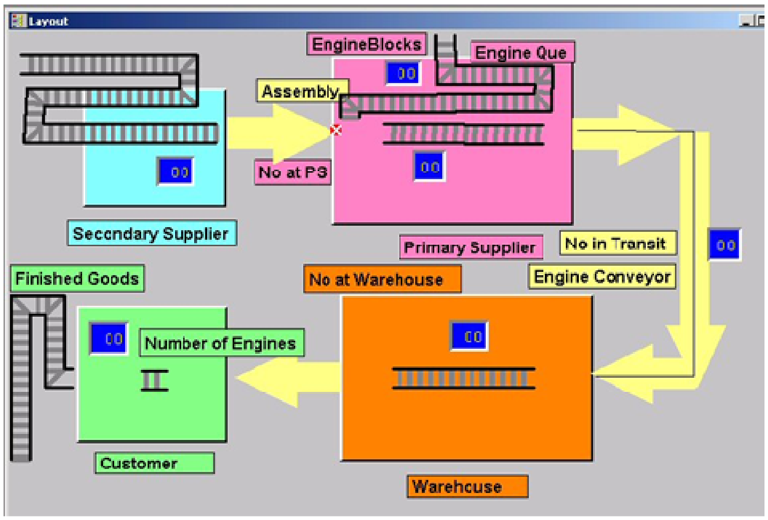

Figure 5. Layout of the supply chain in ProModel

The model uses real-time counters and global variables to define and display the number of parts as they go through the supply chain. The conveyors are long enough to display all parts as they are waiting to be processed. A specified number of cylinders arrives at the secondary supplier with a Poisson distribution. Engine blocks arrive at the primary supplier with another Poisson distribution. One cylinder is assembled with the engine block at the assembly station. The engine block icon is initially grey. After assembly of the cylinder, the color of the engine block changes to blue, indicating an assembled engine. The assembled engine proceeds to the warehouse via the engine conveyor and then on to the customer. The replenishment lead-time is simulated by the travel delay between these stations. For example, if the replenishment lead-time is two months, transportation between these stations takes two months.

The simulation was run with the values of *r* predicted by the base stock model. For example, the base stock model predicted that to obtain a fill rate of 90%, the following inventory levels must be maintained: warehouse-3, primary supplier-5, and secondary supplier-8 for a customer demand of 10 units/yr and a replenishment lead-time of two months. The part counter in this case indicates that 10 engines were delivered to the customer without any backorder. These results are summarized in Table 4.

Table 4. Results from discrete event simulation

Case	Inventory at PS	Inventory at WH	Lead-Time (days)	Engines to Customers	Number of Backorders
1	0	0	60	7	3
2	5	3	60	10	0

Conclusions

The base stock model is effective when the demand is not deterministic and the service factor assumed in the mathematical model is 0.9, which is quite acceptable. The base stock model assumes the replenishment order quantity as 1, and the total inventory cost decreases with replenishment lead-time. The base stock model is beneficial for supply chains having a short replenishment lead-time.

Physical simulation and discrete event simulations are used to validate the results from the base stock model. Both physical simulations and discrete event simulations are designed to include all of the assumptions made by the mathematical model. Hence, all three models are comparable. Demand follows Poisson distribution in both of the simulations. For physical simulation, the backorder cost and inventory holding cost are calculated in each phase of the simulation and summarized in Table 4. We can refer that the total inventory cost is optimum in Phase 2, where the reorder point is the same as that calculated by the mathematical model. In phase I, total inventory cost is more than that of Phase 2 because of backorders. In Phase 3, excess inventory increased the total cost. Thus the values obtained from the mathematical model produce optimal inventory cost. Results from the computer simulation model validate the results predicted by the base stock model.

Results from both the physical and discrete event simulations indicate that these methods can be used to successfully model stochastic systems, like organizational supply chains.

References

[1] Ayers, J. B. (2004). *Supply Chain Project Management*. Delray Beach, FL: St. Lucie Press.

[2] Cooper, R., & Slagmulder, R. (1999). *Supply Chain Development for the Lean Enterprise*. IMA Foundation for Applied Research, Inc. Portland, OR: Productivity, Inc.

[3] Copacino, C. W. (1995). *Supply Chain Management: The Basics and Beyond*. APICS Series on Resource Management. Delray Beach, FL: St. Lucie Press.

[4] Handfield, R. B., and Nichols, Jr., E. L. (1999). *Introduction to Supply Chain Management.* Upper Saddle River, NJ: Prentice-Hall, Inc.

[5] Wallace, J. H., & Spearman, M. L. (1996). *Factory Physics. New York:* McGraw-Hill.

[6] Wincel, J. P. (2004). *Lean Supply Chain Management: A Handbook for Strategic Procurement*. Portland., OR: Productivity Press.
[7] Zheng , Y-S. (1992, January). On Properties of Stochastic Inventory Systems. *Management Science*, *38*(1), 87-103.
[8] Cachon, G. P., & Zipkin, P. (1999). Competitive and Cooperative Inventory Policies in a Two-Stage Supply Chain. *Management Science*, *45(*7), 936-953.

Biography

ALOK K. VERMA is Ray Ferrari Professor and chair of the Engineering Technology Department at Old Dominion University. Dr. Verma received his B.S. in Aeronautical Engineering from IIT, Kanpur, M.S. in Engineering Mechanics, and Ph.D. in Mechanical Engineering from ODU. Prof. Verma is a licensed professional engineer in the state of Virginia, a certified manufacturing engineer, and has certifications in Lean Manufacturing and Six Sigma. He has organized several international conferences as general chair, including ICAM-2006 and ICAM-1999, and also serves as associate editor for three international journals. He serves as editor-in-chief of the *International Journal of Agile Manufacturing*. Dr. Verma's scholarly publications include more than 87 journal articles and papers in conference proceedings and over 50 technical reports. He is actively involved in applied research and has served as a PI or Co-PI on several funded competitive grants exceeding $4.0 million.

He is well known internationally and has been invited to deliver keynote addresses and invited papers at more than 12 national and international conferences on lean/agile manufacturing. Dr. Verma has received the Regional Alumni Award for Excellence for contributions to lean manufacturing research, the International Education Award at ODU and Ben Sparks Medal by the American Society of Mechanical Engineers (ASME). He is active in ASME, American Society for Engineering Education , Society of Manufacturing Engineers, Institute of Industrial Engineers, and the Society of Naval Architects and Marine Engineers. Dr. Verma continues to serve the Hampton Roads community in various leadership positions. He may be reached at averma@odu.edu.